STRUCTURAL AND VISCOUS DAMPING OF BUILDINGS

Slavko ZDRAVKOVIĆ [1], Biljana MLADENOVIĆ[2]

ABSTRACT

The paper analyzes structural, viscous and negative damping. Presence of damping forces causes absorption of energy which gradually reduces amplitudes of oscillations and the motion stops as can be seen from the presented diagrams, except for the negative damping where the oscillations are increased. The negative oscillations occur when the nature of the damping is such that instead of consuming energy from the vibrating system, the energy is added to the system.

The damping forces of oscillating system need not be linear functions of speed or displacement of the moving body.

Keywords: *motion system; energy; structural, viscous and negative damping.*

[1] PhD, Full professor, University of Niš, Faculty of Civil Engineering and Architecture, Aleksandra Medvedeva 14, Niš, Serbia, slavko.zdravkovic@gaf.ni.ac.rs

[2] PhD, assistant professor, University of Niš, Faculty of Civil Engineering and Architecture, Aleksandra Medvedeva 14, Niš, Serbia, biljanam10@gmail.com

1. INTRODUCTION

If mass distribution and the stiffness properties of a structure are known, natural shapes and frequencies can be calculated. This can be done, for example, by solving the equation

$$[D]\{q\} = \frac{1}{\omega^2}\{q\} \tag{1}$$

This matrix equation is a homogeneous set of linear equations. In their deriving, vibrations are considered to last once they are started, without the action of external forces. In reality, such vibrations never occur without reducing the amplitude. The presence of damping forces causes the absorption of energy, which gradually reduces the vibration amplitude and finally stops movement when all the energy originally introduced into the system is absorbed. Therefore, the constant change of potential and kinetic energy, whereby the total energy of the system is maintained at a constant level, applies only to the ideal conservative system. In a non-conservative system where damping forces exist, energy is absorbed (lost) from the system.

If the energy of a non-conservative system is to be maintained at a constant level, an external source must supply energy to the system, in an amount equal to the amount of energy delivered.

Fig.1. shows the dependence of the energy per cycle on the amplitude of a system with one degree of freedom. Curve I represents the energy input from the outside source;

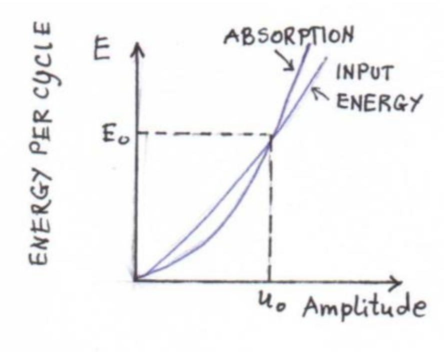

Fig. 1. Dependence of energy per cycle on the amplitude of the system

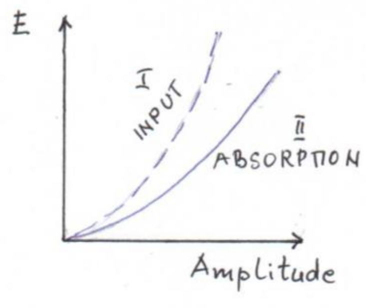

Fig. 2. The input energy always exceeds the absorbed one

curve II represents the absorbed energy. From this figure, it can be seen that for amplitudes less than u_o, the energy input per cycle is greater than that absorbed and, therefore, the amplitude will increase. For amplitudes greater than u_o the absorbed energy is higher and the amplitudes will decrease. At u_o amplitudes, the energy of the system is maintained at some constant level, and the vibrations remain at the maximum constant amplitude.

In Fig.2. the input energy always exceeds the absorbed one, therefore, the equilibrium state is not reached. Amplitudes will progressively increase till the fracture of a structure.

2. THE NATURE OF DAMPING

While mass and stiffness are natural features of the system, damping cannot be such classified. The damping forces may depend on the vibrating system as well as on the elements outside it. The formulation of terms for damping forces is difficult problem that still requires extensive research. The nature of damping is usually described as one of the following:

- Structural damping,
- Viscous damping,
- Coulomb damping,
- Negative damping.

Structural damping is due to internal friction in the material at the joints between the elements of the structure. The resulting damping forces are a function of deformation, or deflection, in the structure. For some elastic system the j-th structural damping force F_{DJ} is proportional to the magnitude of the internal elastic force F_{EJ} and is in the opposite direction with respect to the velocity vector u_j, This dependence is expressed by

$$F_{DJ} = igF_{EJ} \tag{2}$$

where g is a constant, and i is a unit imaginary number.

Viscous damping occurs when a system vibrates in a fluid (air, oil, etc.). Some examples where viscous damping can be observed are shock dampers, a hydraulic damper with air and oil, and the sliding of a body over a greased surface. In viscous damping, the damping force *j* is expressed by

$$F_{DJ} = c_j u_j \tag{3}$$

where the constant c_j is characterized the damping mechanism j. The amplitude of free vibrations at viscous damping decreases exponentially, as shown in Fig. 3.

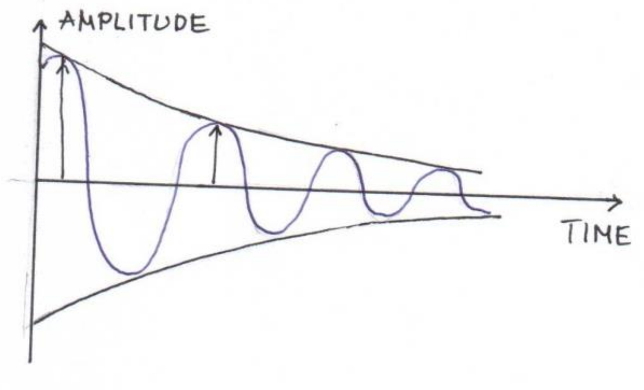

Fig. 3. Amplitude versus time at free vibrations with viscous damping

Coulomb damping, or dry friction, occurs when a body moves on a dry surface. The resulting damping force is approximately constant. It depends on the normal pressure N between the moving body, the surface over which it is moving and the kinetic friction coefficient

$$F_D = \mu N. \tag{4}$$

The amplitude of free vibrations at Coulomb damping decreases linearly, as shown in Fig. 4.

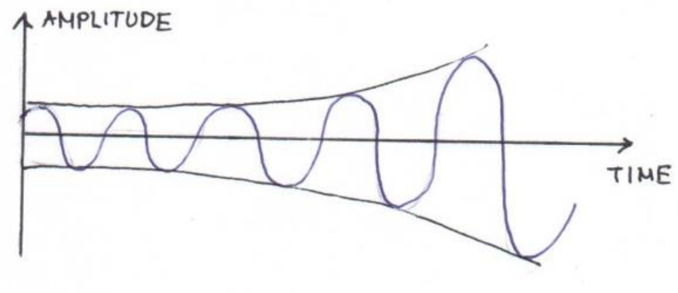

Fig. 4. Amplitude versus time at free vibrations with Coulomb damping

Negative damping occurs when the nature of the damping is such that instead of consuming energy from the vibrating system, energy is added to the system. As an example of negative damping, the cross-section of a wire power line is considered (Fig. 5). At temperatures around 32 ° F, ice is formed around the cross-section of the wire as shown in the figure. When the wind blows to the right, the elongated cross-sectional shape of the wire and ice induces an aerodynamic force F_w on the wire acting in a direction different from the wind direction. During the downward movement of the wire, the air pressure from below is resisted by the force $F_a * p$ ($F_a * p$ = air pressure force). However, if the component acting down F_w is greater than $F_a * p$, the final result is a force in the direction of motion with the addition of energy to the vibrating wire. The similar applies during the upward movement of the wire, whereby energy is added to the system if the component F_w acting upwards is greater than $F_a * p$ d.. Considerations of the air flow around the elongated wire cross section causing certain wind direction, as shown in Fig. 5, is presented in the reference [2]. When negative attenuation occurs, the amplitudes increase progressively, as shown in Fig. 6.

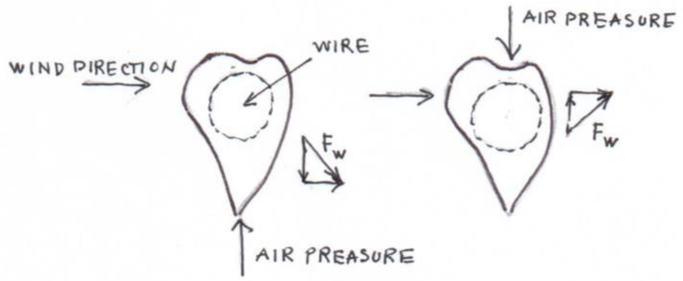

Fig. 5. An example of negative damping

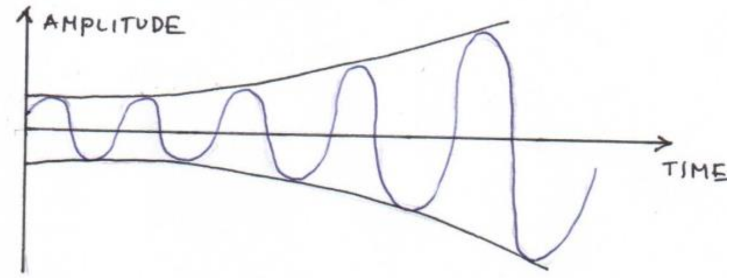

Fig. 6. Amplitude versus time at free vibration with negative damping

The damping forces encountered in vibrating systems need not to be linear functions of the velocity or displacement of the moving body. Thus, for example, experiments show that the air resistance of bodies moving through it at high velocity is approximately proportional to the square of the velocity. In the case of the most real physical systems, it is very difficult to derive an expression for damping forces. In the case of structural or viscous damping, it is not difficult to derive differential equations. It is common (and appropriate), therefore, to replace the damping forces in a system by equivalent viscous damping which causes the same magnitude of energy absorption.

3. LAGRANGIAN DAMPING EQUATIONS

The Lagrange equation in the generalized coordinates q_i $(i = 1,2, \dots \dots, n)$ is expressed by

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\mathrm{dT}}{\mathrm{dq_i}}\right) - \frac{\mathrm{dT}}{\mathrm{dq_i}} + \frac{\mathrm{dU}}{\mathrm{dq_i}} - \mathrm{Q_{Di}} = \mathrm{Q_{Ai}} \tag{5}$$

The term Q_{Di} represents the i^{th} generalized damping force. The expression for Q_{Di} can be derived from the principle of virtual work in terms of the damping forces F_{Dj} $(j = 1,2, \dots.., m)$ in a limited u_j $(j = 1,2, \dots., m)$ coordinate system. The virtual work δW_Dperformed by the damping force F_{Dj}on virtual displacement δ_{uj} u in the coordinate system is given by

$$\delta W_D = \sum_j F_{Dj} \delta_{uj} \tag{6}$$

We now apply coordinate transformation $u_j = u_j\left(q_1, q_2, \dots.., q_n\right) \quad j = 1,2, \dots.., m$

to change over from limited coordinates u to generalized coordinates q. Then it is

$$\delta u_j = \sum_{i=1}^{n} \frac{du_j}{dq_i} \delta q_i \tag{7}$$

and Eq. (7) becomes

$$\delta W_D = \sum_j F_{Dj} \sum_i \frac{du_j}{dq_i} \delta q_i \,. \tag{8}$$

After changing the order of addition and rearrangement, we write

$$\delta W_D = \sum_i \delta_{qi} \sum_j F_{DJ} \frac{du_j}{dq_i} \tag{9}$$

Virtual work δW_D can also be expressed as the sum of the work performed by the generated damping force Q_{Di} on their corresponding virtual displacements δq_i.

By comparing Equations (8) and (9), we write

$$Q_{Di} = \sum_j F_{Dj} \frac{du_j}{dq_i} \quad i = 1,2, \dots \dots, n \tag{10}$$

By a similar procedure, the i^{th} generalized force Q_{Ai} on the right-hand side of equation (5) can be expressed in the form

$$Q_{Ai} = \sum_j F_{Aj} \frac{du_j}{dq_i} \quad i = 1,2, \dots \dots, n \tag{11}$$

where $F_{AJ}(j = 1,2, \dots., m)$ are acting forces in a limited u_j $(j = 1,2, \dots.., m)$ coordinate system.

When the transformation from the coordinates u to the coordinates q is linear, namely, u are linear functions of q we write $\{u\} = [C]\{q\}$,

where is

$$Q_{Ai} = \sum_j F_{Aj} \frac{du_j}{dq_i} \quad i = 1,2, \dots \dots, n$$

$$C_{jk} = \frac{du_j}{dq_k} \quad j = 1,2, \dots., m \quad k = 1,2, \dots.., n$$

With these results, the generalized damping forces and the generalized acting forces expressed by equations (10) and (11), respectively, take the form

$$\{Q\}_D = [C]^T\{F\}_D \tag{12}$$

$$\{Q\}_A = [C]^T\{F\}_A \tag{13}$$

These results agree with the same expressions derived in [5].

4. LAGRANGIAN EQUATIONS FOR STRUCTURAL DAMPING

For the special case of structural damping, the damping forces F_{Dj} are in magnitude proportional to the elastic forces F_{Ej} and are in the opposite direction with respect to the velocities u_j and u in the coordinate system.

$$F_{Dj} = igF_{Ej} \tag{14}$$

Consider, now, that g has the same value for all points of construction. Substituting equation (14) into equation (10), we obtain

$$Q_{Di} = ig\sum_j F_{Ej}\frac{du_j}{dq_i} = igQ_{Ei} = -ig\frac{dU}{dq_i} \tag{15}$$

By a similar procedure as deriving the equation (15), we can show that the ith generalized elastic force Q_{Ei} is given by

$$Q_{Ei} = \sum_j F_{Ej}\frac{du_j}{dq_i} = -\frac{dU}{dq_i} \tag{16}$$

5. EQUATIONS OF MOTION IN THE CASE OF VISCOUSE DAMPING

To formulate the equation of motion in the generalized coordinates q after linear transformation of coordinates

$$\{u\} = [C]\{q\} \tag{17}$$

we form a generalized mass matrix and a damping matrix. They are given in the following form

$$[m]_q = [C]^T[m]_u[C] \tag{18}$$

$$[k]_q = [C]^T[k]_u[C] \tag{19}$$

$$[c]_q = [C]^T[c]_u[C] \tag{20}$$

The similarity of the expressions should again be noted. Generalized acting forces are expressed by equation (13). Then the equations of motion are written directly as

$$[m]_q\{\ddot{q}\} + [c]_q\{\dot{q}\} + [k]_q\{q\} = \{Q\}_A \tag{21}$$

When similar forces Q_{Ei}, derived from some potential U, we conclude from (5) that for some system in which energy is absorbed due to structural damping, the i^{th} Lagrange equation in the generalized coordinates q takes the form

$$\frac{d}{dt}\left(\frac{dT}{d\dot{q}_i}\right) - \frac{dT}{dq_i} + (1+ig)\frac{dU}{dq_i} = Q_{Ai} \qquad i = 1,2,\ldots,n \tag{22}$$

6. CONCLUSION

Differential equations of motion for a system with structural or viscous damping are given by

$$[m]\{\ddot{q}\} + (1+ig)[k]\{q\} = \{Q\}_A \tag{23}$$

and

$$[m]\{\ddot{q}\} + [c]\{\dot{q}\} + [k]\{q\} = \{Q\}_A \tag{24}$$

These two equations represent the general formulation of a wide range of problems in structural dynamics. The acting forces $\{Q_A\}$ may be: zero, harmonic, periodic, aperiodic and random.

Each type of force can act on some system of construction in which there is damping or damping is considered insignificant.

ACKNOWLEDGEMENT

This research is conducted at The Faculty of Civil Engineering and Architecture of University of Niš in the framework of the project in the field of technological development in the period 2011-2019, and titled Experimental and theoretical investigation of frames and plates with semi-rigid connections from the view of the second order theory and stability analysis (TR 36016), financially supported by the Ministry of Education and Science of the Republic of Serbia.

REFERENCES

[1] Ančić D., Fajfar P., Petrović B., Szavitsnossan A., Tomažević M. Earthquake engineering, Građevinska knjiga, Beograd, 1990. (in Serbian)

[2] Den Hartog, J. P. : Vibracije u mašinstvu, Građevinska knjiga, Beograd 1972. (translated from English)

[3] Zdravković Slavko. Dynamics of structures with earthquake engineering,, Građevinsko-arhitektonski fakultet u Nišu, AGM knjiga Beograd, 2017. (in Serbian)

[4] R. W. Clough, J . Penzien: Dynamics of structures, McGraw Hill, New York, 1975.

[5] V. Harti, M . Rubinstain: Dinamika konstrukcija, BIGZ, Beograd 1973.

[6] Zlatkov D., Zdravković S., Stanojev M., Proporcionalno prigušenje sa posebnim aspektom na zaleđeni žičani vod, 8. međunarodno savetovanje -Rizik i bezbedonosni inženjering, Kopaonik, Visoka Tehnička škola strukovnih studija u Novom Sadu,1SBN 978-86-6211-057-2 COBISS-SR-ID 276609031, 2013.