ANALYTICAL METHOD ACCORDING (AAEM) OF BAŽANT VERSUS NUMERICAL METHOD OF VOLTERRA IN ANALYSIS OF STATICALLY INDETERMINATE COMPOSITE STEEL-CONCRETE BEAMS REGARDING CREEP OF CONCRETE

CHAVDAR STOYANOV [1], DONCHO PARTOV[2], VLADIMÍR KŘÍSTEK [3]

ABSTRACT

The paper presents analytical method, according (AAEM) of Bažant for analysis of the stress changes due to creep in statically indeterminate composite steel-concrete beam. Creep have a considerable impact upon the performance of composite beams, causing increased deflection as well as affecting stress distribution. Creep analysis of composite structures is normally performed on the basis of the linear theory of viscoelasticity for aging materials. In general, time-dependent deformation of concrete regarding creep phenomena may severely affect the serviceability, durability and stability of structures. For determining the relaxation of a bending moment, aroused over the support due to its settlement, in a continuous composite steel- concrete beam with respect to time "t", system of two independent algabraical equations have been derived, on the basis of the theory of the viscoelastic body of Arutyunyan–Trost-Bažant. Example with the model proposed is investigated. The results obtained from analytical methods is compared with the results from numerical analysis.The creep functions is suggested by the "CEB-FIP" models code 2010. The elastic modulus of concrete $E_c(t)$ is assumed to be constant in time 't'.

Key words: *relaxation, numerical analysis, indeterminate composite steel-concrete beam Volterra integral equations, rheology, AAEM method*

1. Msc. PhD student,1373 Sofia,Suhodolska Str.175,Univ.Str.Eng. and Arch, ing4040@abv.bg

2. Prof. PhD, FEng, 1373 Sofia, Suhodolska Str.175, Univ. Str. Eng. and Arch., partov@vsu.bg

[3]Prof. DSc, FEng, D-r h. c., FCE, Dept. of Concrete and Masonry Structures, Thákurova 7, 166 29 Prague 6, Czech Republic, VladimirKristek@seznam.cz

1.INTRODUCTION

Steel-concrete composite beams are wide spread form of construction in both buildings and bridges. One of the very serious problem in the steel-concrete composite beams is to determine, their time-varying behaviour under sustained service loads, which drawn the attention of engineers who were dealing with the problems of their design more than 60 years[2,3,6,7,9,13]. A large number of practical problems concerning the influence of creep effect on the reliability and durability of concrete and composite structures can be solved exactly, through the four fundamental theorems of this theory, as demonstrated by the (Chiorino, 2010)[5]. The investigation of creep effect in composite steel-concrete beams has been an important task for engineers since the first formulation of the mathematical model of linear viscoelasticity(Bažant, 1975 and Branson, 1977)[5]. In this paper it is analyzed the time dependent behavior of statically indeterminate composite steel-concrete beam with respect to rheological properties of concrete according to world code provisions: Eurocode2(Comité Européen de Normalisation [CEN] 2004a[1].

2. **ABOUT SOME METHODS FOR TIME-DEPENDENT ANALYSIS**

2.1. Formulation of the age-adjusted effective modulus method

However, **in order to avoid the mathematical problems** in solving of the integral equations of Volterra for treating the problem connected with the creep of concrete structures, it has been revised the integral relationship into new algebraic stress-strain relationship :

$$\varepsilon_{ct} = \frac{\sigma_{c0}}{E_{c0}}\left[1+\varphi_t\right] + \frac{\sigma_{ct}-\sigma_{c0}}{E_{c0}}\left[1+\rho\varphi_t\right], \tag{1}$$

Where: ρ is the relaxation coefficient known from Trost-Zerna works[16,17]. When more extensive test date and data of long duration became available and were systematically analysed from Bažant[4,8], it turned out that the afore-mentioned theory leading to differential equations are overall not more accurate than the effective modulus methods(Partov and Kantchev[10,11,12]), which leads to algebraic linear equations with respect to time *t*. According Bažant and Jirasek[8] none of them is sufficiently accurate compared to the computer solutions for e realistic (un - simplified) compliance function based on long – time –measurements with a broad range of ages at loading. A remedy that is sufficiently accurate in most basic situations we can found in the ***age-adjusted effective modulus method***, proposed and mathematically proven by Bažant[8], as a modification and refinement of the relaxation method, semi-empirically developed by Trost[14,15,16,17]. By using algebraic approach a new simpler forms for (1) are obtained from Bažant[8]. His methods are based on the hypothesis that the strain in the concrete fibers can be considered as a linear function of the creep coefficient. This permits transforming (1) in to (2):

$$\begin{aligned}\varepsilon_c(t,t_0) &= \varepsilon^{sh}(t) + \sigma_c(t_0)\left[\frac{1}{E_c(t_0)} + \frac{\varphi_{28}(t,t_0)}{E_{c28}}\right] + \\ &+\left[\sigma_c(t)-\sigma_c(t_0)\right]\left[\frac{1}{E_c(t_0)} + \frac{\chi(t,t_0)\varphi_{28}(t,t_0)}{E_{c28}}\right]\end{aligned} \tag{2}$$

where: $$\chi(t,t_1) = \frac{E(t_1)}{E(t_1)-R(t,t_1)} - \frac{1}{\varphi(t,t_1)}; \tag{3}$$

and: $$R(t,t_1) = \frac{0{,}992}{J(t,t_1)} - \frac{0{,}115}{J(t,t-1)}\left[\frac{J(t-\Delta,t_1)}{J(t,t_1+\Delta)}-1\right];\ \Delta = \frac{t-t_1}{2}; \tag{4}$$

where: $\chi(t,t_0)$ cs the aging coefficient; $\varphi(t,t_0)$ - the creep coefficient ; $R(t,t_0)$ - relaxation function, i.e., the stress response to a constant unit strain applied at the time t_0; E_c - the elastic modulus of concrete at 28 days. The age-adjusted effective method (AAEM) directly assumed the expression provided by (2) for the aging coefficient. In this case, it is necessary to evaluate previously the relaxation function $R(t,t_0)$. This function is calculated numerically by applying the step-by-step procedure of the general method to the integral type relation between the creep and the relaxation function. For the computation of the values of the χ aging coefficient at any desired time t and for any desired value of influencing parameters, on the base of prediction models CEB90, GL2000 and B3 a software tool is available at www.polito.it/creepanalysis/. The main advantage of the method, consequent to the adoption of the algebraic formulation instead of the **hereditary integral formulation** for the constitutive viscoelastic equations of the different parts, consists in avoiding the need to store the time history of each sub-element. The AAEM algebraic simplification of the hereditary integral constitutive law may be adopted in the frame of the force(compatibility) or deformation(equilibrium) methods for the analysis of the effects of creep and shrinkage on the overall response of structures. The AAEM methods is frequently adopted for the basic investigation of stress redistribution in non-homogenous cross-section, like prestressed concrete section with prestressing and reinforcing steel in one or multiple layers, and steel-concrete composite section. One other advantage of the AAEM method is the fact that $\chi(t,t')$ varies relatively little with the age t' for sufficiently long elapsed times. Its long-term values are in a range between 0,5 and 1,0, the most common values, for typical values of t' and other influencing parameters, being contained in a narrower range between 0,7 and 0,9, in particular for B3 and GL2000 creep prediction models. Therefore, considering in uncertainties in creep prediction and the fact that the aging linear visco-elasticity approach on which the aging coefficient is based is also only an approximation, the adoption of a fixed long-term value for the aging coefficient comprised in this narrower range, independently of creep properties of the structural element being considered, leads often to satisfactory accuracies in the evaluation of the long-term structural responses, particularly in the conceptual and preliminary design stages and in the assessment of structures of low sensitivity to time –dependent effect. It is often adequate to use the value $\chi = 0,8$. In the papers [10,11] using the mathematical model in Fig.3, the authors introduce the system of linear Volterra integral equation of second kind and obtain the results from their numerical solutions. The kernels of the integral equations contain the respective creep functions corresponding to the model EC2 [1].

3. RELAXATION OF A SUPPORTS MOMENT IN COMPOSITE BEAM

3.1. General date

In statically indeterminate composite steel-concrete beams, a new equilibrium state arises in the case of a kinematic changes of the construction. This phenomenon can be observed by the settlement of an internal support, or by impact of forced mounting movement of the support in order to introduce prestressing in the building structure. Thanks to the visco-elastic character of the concrete, redistribution of the internal sectional forces in the beam has begun, for which a mechanic- mathematical model is constructed. It is known that in the case of composite beams, with a constant moment of inertia along its length, the new elastic line induced by the settlement of the support remains unchanged at time t. This fact has been proven in the study of a homogeneous concrete beam by settlement of the support. This practical example is evidenced by the presence of the second theorem by G. Colonnetti(1941)[4]. From the point of view of hereditary mechanics, it is a purely relaxing process in which the deformation introduced remains constant during the instantaneous settlement of the support. Thus, the deformation introduced in the composite beam construction creates in it a stress state, expressed by the internal forces in the composite structure. In the preserved form of the elastic line after the lowering of the support and the emergence of the new support moment $M_B = 3(EI/l^2).\Delta$: in time there is no change in the curvature of the steel beam, which unconditionally imposes the condition: $M_{st,\varphi} = 0$.

3.2. Example according EC4: Continuous beam by the settlement of an internal support

Let us try to apply this theory to a continuous composite steel-concrete beam with three supports, subjected to settlement of mean supports, in which we import by lowering of the mean support with $\Delta = 22{,}58cm$, a prestressing moment with a value: $M_0 = \Delta 3 E_{st} I_i / l^2 = 11050 kNm$ *(Fig.1)*.

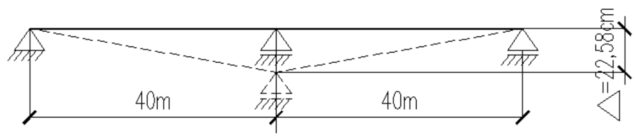


Fig. 1. Statically indeterminate composite steel-reinforced concrete structures subjected to kinematic support,s settlement

Let,s a continuous composite steel- concrete beams with three supports has two equal openings equal to 40 m. The cross-section of the bridge beam is as follows in millimeters: bottom flange: -60x500; stem: -12x2000; upper flange: 20x300; thickness of reinforced concrete slab: 200mm; effective plate width: 3000 mm(Fig.2,3).

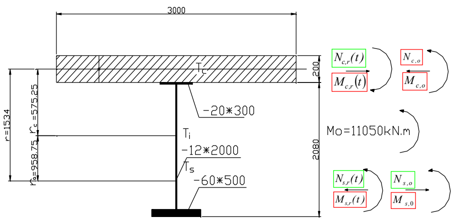


Fig. 2: Composite beam

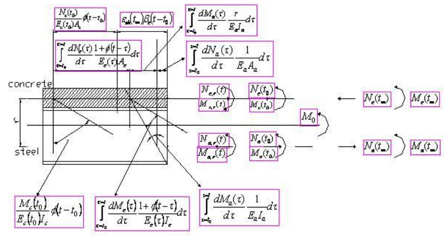


Fig.3. Mechanic-Mathematical model

4. INVESTIGATION OF A CONTINUOUS BEAM BY SETTLEMENT OF THE SUPPORT USING INTEGRAL AND ALGABRAIC METHODS

In this paper the authors treat the investigation of the stress-strain state of a continuous composite beam by settlement of supports regarding the creep of concrete, which has been described with the functions of the "CEB-FIP" models code 2010. Till now, these functions have been applied in the EC4 to calculate the stress-strain state of composite constructions in time t using the "Effective Modulus Method"(EMM). In order to answer some of the issues discussed on the pages of the international journals about the advantages and disadvantages of the integral and algebraized concept of expressing the process of creeping the concrete over time as scientific concepts, a new mechanics-mathematical algebraic model for determination of the forces in statically indeterminate composite steel-composite constructions, in case of settlement of the supports, taking into account the influence of concrete creep, according to the model of **"**CEB-FIP" models code 2010. The main hypotheses in the elastic analysis of composite steel-concrete sections with stiff (rigid) shear connectors are assumed as in the previous papers[10,11,12].

4.1. Basic equation of equilibrium

Let us denote both the normal forces and the bending moments in the cross-section of the plate and the girder after the loading in the time $t = 0$ with $N_{c,0}$, $M_{c,0}$, $N_{a,0}$, $M_{a,0}$ and with $N_{c,r}(t)$, $M_{c,r}(t)$, $N_{a,r}(t)$, $M_{a,r}(t)$ a new group of normal forces and bending moments, arising due to creep and shrinkage of concrete. For a composite bridge girder with $J_c = \frac{A_c (n I_c) n}{A_s I_s} \leq 0.2$ according to the suggestion of (Sonntag 1951) we can write the equilibrium conditions in time t as follows:

$N_{c,r}(t) = N_{a,r}(t)$; **(5);** $\quad M_{c,r}(t) + N_{c,r}(t) r = M_{a,r}(t)$; which don't arise; (6)

Due to the fact that the problem is a twice internally statically indeterminate system, the equilibrium equations (1), (2) are not sufficient to solve it. It is necessary to produce two additional equations in the sense of compatibility of deformations of both steel girder and concrete slab in time *t* (Fig.3).

4.2. Solving the problem with settlement of the support using the integral equations

Using the mathematical formulas for expressing creep according to the Eurocode 2 model, for determining of the internal forces in the statically indeterminate composite beam in time "t", a system of two independent integral equations of the second order of Volterra will be derived[10,11,12]. The solution will be made using our knowledge of the study of the distribution of forces from the concrete slab to the steel beam, as by an constant external moment. The difference will only consist in the fact, that while in the cross-section of the simple beam the sum of the internal forces, during the creep process, will again be equal to the external constant bending moment, then by the settlement of support, the aroused external moment will relax, i.e. decreases. Its value will be obtained as the particular cross-section forces, aroused as a result of the creep of concrete, transformed into a bending moment, must be removed from the moment over the support, obtained from its settlement. The assignment of the task is as follows: the resulting momentum due to the sudden displacement of the support starts to decrease. For this purpose, the particular cross-section internal forces in concrete and steel are determined at time t = 0. They decrease in $t = \propto$ to a certain value. The values obtained are in equilibrium and it is possible to determine the resultant moment over the support in the beam. The same Volterra equations are recorded as in simple beams, taking into account the absence of a moment in the time over the support: $M_{st,\varphi} = 0$, because the task is considered in the light of the second Colonnetti theorem, which hypothesis is transformed into the adoption of an endlessly stiff steel beam, in the process of creep, that explane the proposition, that elastic line of the beam does not change, but only the affine is moved in relation to the initial shape, i.e. no change in curvature and which makes the condition for: $M_{st,\varphi} = 0$. Note: For the instant support settlement, account must be taken of the behavior of the system subject to the second Colonnetti, where the component: $M_{a,r}(t) = 0$. Considering the above, the relaxation in the instant displacement of the support can be covered by the emergence of a new, equilibrated group of cross-section forces in time $t = \propto$, as follow in next text.

4.3. Deriving of the generalised mechanic-mathematical model using integral equation

Using the mentioned approach in [10,11,12] for constant elasticity module of concrete for assessment of normal forces $N_{c,r}(t)$ and bending moment $M_{c,r}(t)$ two linear integral Volterra equations of the second kind are derived.

$$N_{c,r}(t) = \lambda_N \int_{t_0}^{t} N_{c,r}(\tau) \frac{d}{d\tau} \left[\left[1 + \phi_{RH} \beta(f_{cm}) \beta(\tau) \beta_c(t-\tau) \right] \right] d\tau + + \lambda_N(t) N_{c,0} \phi_{RH} \beta(f_{cm}) \beta(t_0) \beta_c(t-t_0) \tag{7}$$

$$M_{c,r}(t) = \lambda_M(t) \int_{t_0}^{t} M_{c,r}(\tau) \frac{d}{d\tau} \left(\left[1 + \phi_{RH} \beta(f_{cm}) \beta(\tau) \beta_c(t-\tau) \right] \right) d\tau + \lambda_M(t) M_{c,0} \phi_{RH} \beta(f_{cm}) \beta(t_0) \beta_c(t-t_0) \tag{8}$$

Where $\lambda_N = [1 + E_c A_c / E_a A]^{-1} = 0,375$. In each of these equations the functions:
$N_{c,0} \phi_{RH} \beta(f_{cm}) \beta(t_0) \beta_c(t-t_0)$; $M_{c,0} \phi_{RH} \beta(f_{cm}) \beta(t_0) \beta_c(t-t_0)$; $\frac{d}{d\tau}[1 + \phi_{RH} \beta(f_{cm}) \beta(\tau) \beta_c(t-\tau)]$ are given and

$\lambda_M = 1$. The final result of the of the prestressed moment over the support after its relaxation is obtained by subtracting the work from the relaxed normal force multiplied by the shoulder to the center of gravity of the composite beam and the value of the relaxed moment in the concrete slab.
$M(t = \propto) = M_0 - N_{c,r}(t_\propto).r - M_{c,r}(t_\propto) = 1105000 - 239862,3.1,534 - 2200,7 = 7348,50 kNm. > 6869,56 kNm(EMM)$.

4.4. Deriving of the generalised mechanic-mathematical model using algebraic equation according aaem methods of Bažant

Using the above mentioned approach, for constant elasticity module of concrete for assessment of normal forces $N_c(t)$ and bending moment $M_c(t)$ two algebraic expressions are derived:

Calculating: $N_c(t)$ using the formulae: $N_c(t) = \dfrac{N_c(t,t_0).\varphi(t,t_0).\lambda_N}{1+\chi(t,t_0).\varphi(t,t_0)\lambda_N}$, (according Bažant) (9)

Calculating: $M_c(t)$ using the formulae: $M_c(t) = \dfrac{M_c(t_0).\varphi(t,t_0)\lambda_M}{1+\chi(t,t_0).\varphi(t,t_0)\lambda_M}$, (according Bažant) (10)

4.5. Numerical method of solution

The integral equations (9) and (10) are weakly singular Volterra integral equation of the second kind. To solve (9) and (10) we use the method called product trapezoidal rule(Atkinson,1997)[10,11,12].

5. NUMERICAL EXMPLE

The two methods presented in the previous paragraph is now applied to a continuous composite steel-concrete beam with three supports, subjected to settlement of mean supports, whose cross section is shown in Fig. 2. The following parameters are chosen according EC2 model[1].

$h_0 = 2AC/u = \dfrac{2.3000.200}{3000+3000} = 200\ mm$; $\beta_H = 150\left[1+(1.2*80/100)^{18}\right]h_0/100+250 = 693,881 < 1500;$;

$\beta(f_{cm}) = \left.\dfrac{5.3}{(f_{cm}/10)^{0.5}}\right|_{f_{cm}=30} = 3.06$; $\beta(t_0) = \left.\dfrac{1}{0.1+(t_0)^{0.2}}\right|_{t_0=28} = 0,488449545$; $\phi_{RH} = 1+\left.\dfrac{1-RH/100}{0.46\sqrt[3]{(h_0/100)}}\right|_{RH=80,h_0=200} = 1,3451$; $\phi_0 = \phi_{RH}\beta(f_{cm})\beta(t_0) = 2,0104$;

$\beta_c(36500-28) = 0,9943$; $\phi_{t=36500} = \phi_0\beta_c(36500-28) = 2,01.0,99 = 1,99 = 2,0$;

$\beta(\tau) = \dfrac{1}{0.1+(\tau)^{0.2}}$; $\beta_c(t-\tau) = \left[\dfrac{t-\tau}{\beta_H+(t-\tau)}\right]^{0.3}$.

$$E_c = 3,5.10^4\ MPa,\ E_a = 2,1.10^5\ MPa,\ A_c = 6000\,cm^2,\ A_a = 600\,cm^2, n = \frac{E_a}{E_c} = 6,00$$

$$I_c = 200000\,cm^4,\ I_a = 3567300\,cm^4,\ r_c = 57,525\,cm,\ r_a = 95,875\,cm,\ r = 153,4\,cm,$$

$$A_i = 1600\,cm^2,\ I_i = 12425000\,cm^4.M_0 = 11050\,kNm),\ N_{c,o} = N_{a,o} = \frac{A_a}{I_i}r_a.M_o = 5115,90\,kN,$$

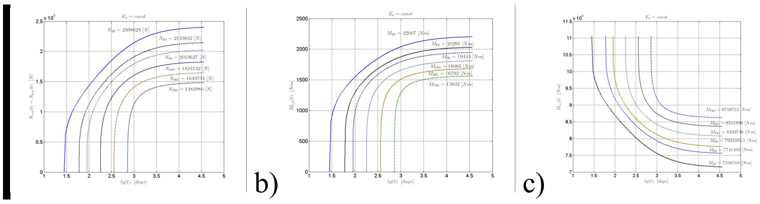
$$M_{c,o} = 29,644\,kNm; M_{a,o} = \frac{I_a}{I_i}M_o = 3172,528kNm,$$

5.1. Numerical solution using integral equation of Volterra

In Fig. 4a,b, c, is shown the values of normal forces and bending moments in time $t \propto$ equal 100 years. A numerical method for time-dependent analysis of composite steel-concrete sections according EC2[1] models is develop using MATLAB code and numerical algorithm, which was successfully applied to a continuous composite steel-concrete beam. For a good accuracy of the time values, the numerical results are presented on logarithmic time scales. The choice of the length of time step(one day) of the proposed numerical algorithm is based on numerous numerical experiments with different steps. So we conclude that good results can be achieved from practical point of view with one day step. For our purpose we consider a period of about 36500 days(100 years).

5.2. Force histories in support section of the beam according the received numerical results

In the concrete plate the normal component: $N_c(t_0)$ and the bending moment: $M_c(t_0)$ decrease by effect of creep. In the steel beam, only the normal component $N_a(t_\infty)$ arise, which increase the total negative stresses in the upper flange of the beam and decrease the total positive stress in the bottom flange of the beam. The decrease of the stresses in concrete slab is accompanied by a gradual migration of stresses from the concrete slab to the steel beam. Consequently, the stress history in the composite beam becomes the most interesting aspect of this study. These graphs also shows how important is the age of concrete by the process of the relaxation. Considering the value of the normal component: $N_c(t_0)$ and the bending moment: $M_c(t_0)$ we see that they decrease more for young concrete and a little for old one. Above all the influence of concrete age at loading time t_0 is significant only when its values are very low (i.e. with young concrete).



a) b) c)

Fig. 4a,b,c. Normal forces $N_{c,r}(t) = N_{a,r}(t)$, bending moments $M_{c,r}(t)$ and $M_{0,\infty}(t)$ in time: $t_\infty = 36528 - 337230$ days, when relaxation begin in time t$_o$ = 28, 60, 90, 180, 365 and 730 days

The results obtained by numerical method, using the integral equation of Volterra, on the basis of the theory of the viscoelastic body of Maslov- Arutyunyan–Trost-Bažant and according the creep functions, suggested by the "CEB-FIP" models code 2010[1], are comparable with the results obtain by the AAEMM, based on the Bažant proposal(See table: 1).

6. DETERMINING OF THE AGING COEFFICIENT: $\chi(t,t_0)$; THE CREEP COEFFICIENT $\varphi(t,t_0)$ AND RELAXATION FUNCTION $R(t,t_0)$ ACCORDING AAEMM OF BAŽANT.

6.1. Determining the coefficient of creep using AAEMM of Bažant

$t_0 = 28$ *days;* $t_\infty = 36500$ *days;* $E_c(t_0) = 35000 MPa$ *;*

The creep coefficient:

6.1.1. $\phi(t = 36500, t_0 = 28) = \phi_{RH}\beta(f_{cm})\beta(t_0)\beta_c(t - t_0) = 1{,}3451.3{,}06.\dfrac{1}{0{,}1 + (t_0 = 28)^{0{,}2}}\left[\dfrac{t - t_0 = 36500 - 28 = 36472}{\beta_H + (t - t_0) = 26465{,}82}\right]^{0{,}3} =$

$= 1{,}3451.3{,}06.0{,}488449545.0{,}994362065 = 1{,}9991276408$

6.1.2. $J(t = 36500, t_0 = 28) = \dfrac{1 + \varphi(t,t_0)}{E_{cmt_0}} = \dfrac{1 + 2.00}{35000} = 0{,}000085714$;

6.2.1. $\varphi(t, t_0 = (t-1)) = (36500, 36499) = 1{,}3451.3.06\dfrac{1}{0{,}1 + (36499)^{0{,}2}}\left[\dfrac{t - t_0 = 36500 - 36499 = 1}{\beta_H + (t - t_0) = 693{,}88 + 1}\right]^{0{,}3} =$

$= 1{,}3451.3{,}06.0{,}120854599.0{,}160419381 = 0{,}069849972;$

6.2.2. $J(t = 36500, t - 1 = 36499) = \dfrac{1 + \varphi(t, t-1)}{E_{cmt_0}} = \dfrac{1 + 0{,}069849972}{35000} = 0{,}000030567$;

6.2.2.1. $\Delta = \dfrac{t - t_1}{2} = \dfrac{36500 - 28}{2} = 18236$;

6.3.1 $\varphi(t-\Delta,t_1)=\varphi(36500-18236,28)=\varphi(18264,28)=1,3451.3,06.\frac{1}{0,1+28^{0,2}}\left[\frac{t-t_0=18264-28=18236}{693,88+18236}\right]^{0,3}=$

$=1,3451.3,06.0.4488449545.0,988859332=1,988063378;$

63.2. $J(t-\Delta,t_1)=\frac{1+\varphi(t-\Delta,t_1)}{E_{cmt_0}}=\frac{1+1,9880633378}{35000}=0,000085373;$

6.4.1. $\phi(t,t_1+\Delta)=\varphi(t=36500,t_1=28+18236=18264)=$

$=1,3451.3,06.\frac{1}{0,1+18264^{0,2}}\left[\frac{36500-18264=18236}{693,88+18236}\right]^{0,3}=;$

$=1,3451.3,06.0,13855474.0,988859332=0,563938706$

6.4.2. $J(t,t_1+\Delta)=\frac{1+\varphi(t,t_1+\Delta)}{E_{t_0}}=\frac{1+0,563938706}{35000}=0,000044683;$

6.5. $R(t,t_0)$**- relaxation function:**

$$R(t,t_0)=\frac{0,992}{J(t,t_0)}-\frac{0,115}{J(t,t-1)}\left[\frac{J(t-\Delta,t_1)}{J(t,t_1+\Delta)}-1\right]=$$

$$\frac{0,992}{0,000085714}-\frac{0,115}{0,000030567}\left[\frac{0,000085383}{0,000044683}-1\right]=8147,348001$$

6.6. The aging coefficient

$\chi(t,t_0)=\chi(36500,28)=\frac{E(t_0)}{E(t_0)-R(t,t_0)}-\frac{1}{\phi(t,t_0)}=\frac{35000}{35000-8147,34800}-\frac{1}{2}=0,803409436$;

6.7. $N_c(t)=\frac{N_c(t,t_0).\varphi(t,t_0).\lambda_N}{1+\chi(t,t_0).\varphi(t,t_0)\lambda_N}=\frac{511,580.2,0.0,375}{1+0,803409436.2,0.0,375}=\frac{383,685}{1,602557077}=239,420Mp;$

(accordingAAEMM); Where: $\lambda_N=\frac{1}{1+\frac{E_cF_c}{E_{st}F_{st}}}=0.375$; $\lambda_N=\frac{1}{1+\frac{350000.6000}{2100000.600}}=$

$$\lambda_N=\frac{1}{1+\frac{10}{6}}=0,375.$$

6.8. Calculating $M_c(t)$ **using the formulae (10): (according Bažant)**

$M_c(t)=\frac{M_c(t_0).\varphi(t,t_0)\lambda_M}{1+\chi(t,t_0).\varphi(t,t_0)\lambda_M}=\frac{2,9644.2,0.0,99074}{1+0,803409436.2,0.0,99074}=\frac{5,873899312}{1+1,5919}=2,266252Mpm.$(according

Bažant) , by $0,99074=\lambda_M$; Calculating $M\ (t\propto)$ over the support using the formulae:

$M\ (t\propto)=M_0-N_c(t).r-M_c(t)=1105-239,420.1,534-2,266=735,46Mpm$,(according AAEMM);

$M_a(t)=M_c(t)+N_c(t).r=1590,102+7874,463777.1,0387=9769,307525daN$ daNm.

Comparisons between the results obtained from the numerical solution and AAEM methods are as follows: $N_c(t)$ =7874,463 daN (by AAEM) and $N_c(t)$ =7896,10 daN (by numerical method); (Δ=0,274%). $M_c(t)$ =1590,102 daNm (by AAEM) and $M_c(t)$ = 1550,40 daNm (by numerical method) (Δ=2,490%).

Table 1. The stresses in upper and lower fibers of concrete plate and steel beam in time t_∞ according Integral equation of Volterra and algebraic equation Bažant

Stress in time t_∞ NUMERICAL METHOD	$t_0 = 60$ days	Stress in time t_∞ AAEMM(Bažant)	$t_\infty = 25610$ days
M_0 (kNm)	1237	M_0 (kNm)	1237
$\sigma_c^{up} = -\frac{N_c(t_\infty)}{A_c} \mp \frac{M_c(t_\infty)}{I_c} z_c$ (MPa)	-1,1387	$\sigma_c^{up} = -\frac{N_c(t_\infty)}{A_c} \mp \frac{M_c(t_\infty)}{I_c} z_c$ (MPa)	-1,1319
$\sigma_c^{down} = -\frac{N_c(t_\infty)}{A_c} \mp \frac{M_c(t_\infty)}{I_c} z_c$ (MPa)	-0,596	$\sigma_c^{down} = -\frac{N_c(t_\infty)}{A_c} \mp \frac{M_c(t_\infty)}{I_c} z_c$ (MPa)	-0,604
$\sigma_a^{up} = \frac{N_c(t_\infty)}{A_a} \mp \frac{M_a(t_\infty)}{I_a} z_a$ (MPa)	-11,36	$\sigma_a^{up} = \frac{N_c(t_\infty)}{A_a} \mp \frac{M_a(t_\infty)}{I_a} z_a$ (MPa)	-11,37
$\sigma_a^{down} = \frac{N_c(t_\infty)}{A_a} \mp \frac{M_a(t_\infty)}{I_a} z_a$ (MPa)	41,60	$\sigma_a^{down} = \frac{N_c(t_\infty)}{A_a} \mp \frac{M_a(t_\infty)}{I_a} z_a$ (MPa)	41,62

ACKNOWLEDGMENTS
The authors would like to thank to "Ministry of Education" in Sofia for Research Foment.

CONSLUSION

The age-adjusted effective modulus method must to know that is development to be theoretically exact for any creep function deriving in EC2, ACI209R-92 and G&L model. In our case to solve the creep problem in the composite steel-concrete beams, we used the EC2 model for describing the creep evaluation. The results obtained by the AAEM method of Bažant are completely comparable with the results based on numerical method according to the EC2 provision and EMM of EC4.

REFERENCES

[1] ACI 209.2R-08, Guide for Modeling and Calculation of Shrinkage and Creep in Hardened Concrete, American Concrete Institute, ACI 209.2R-08, ACI, (2008), 48 pp.

[2] Alexandrovskii S. V. (1966), Analysis of Plain and Reinforced Concrete Structures for Temperature and Moisture Effects (with Account of Creep) (in Russian), Stroyizdat, Moscow, (1966), pp 443.

[3] Arutyunian N. Kh., *Some Problems in the Theory of Creep* (in Russian), Techteroizdat, Moscow, (1952) (French transl., Eyrolles 1957, English transl., Pergamon Press 1966), pp 319.

[4] Bažant Ž. P., Editor , *Mathematical Modeling of Creep and Shrinkage of Concrete*, John Wiley & Sons, (1988), pp 459.

[5] Chiorino M. A., An Internationally harmonized Format for Time dependent Analysis of Concrete Structures, Proceedings IABSE-FIP Conference Dubrovnik,(2010), Vol.1, pp.473-480.

[6] Dischinger F., Untersuchungen über die Knicksichereit, die Elastische Verformung und das Kriechen des Betons bei Bogenbrücken, *Der Bauingenieur*, Vol.18, (1937), pp. 487-520, 539-52, 595-621.

[7] [55] Dischinger F., Elastische und Plastische Verformungen der Eisenbetontragwerke und Insbesondere der Bogenbrücken , *Der Bauingenieur*, Vol.20, (1939), pp. 53-63, 286-94, 426-37, 563-72.

[8] Jirasek M. and Bažant Z.P. (2002), *Inelastic Analysis of Structures*, J.Wiley & Sons, (2002), 734 pp.

[9] Maslov G. N., Thermal Stress States in Concrete Masses, with Account of Concrete Creep (in Russian), Izvestia NIIG, 28, (1941), pp 175-188.

[10] Partov D., Kantchev V., „Time-dependent analysis of composite steel-concrete beams using integral equation of Volterra, according EUROCODE-4“, *Engineering MECHANICS, Vol. 16, 2009, No 5, pp 367-392.*

[11] [11] Partov D., Kantchev V., „Level of creep sensitivity in composite s steel-concrete beams, according to ACI 209R-92 model, comparison with EUROCODE-4(CEB MC90-99)“, *Engineering MECHANICS, Vol. 18, 2011, No 2, pp 91-116.*

[12] [12] PartovD., Kantchev V., "Gardner and Lockman model (2000) in Creep analysis of composite steel-concrete section ", *ACI Structural Journal*, Vol.111, No. 1(January-February), 2014, pp 59-69.

[13] [13] Prokopovich I. E. Fundamental study on application of linear theory of creep, (In Russian),Vyssha shkola, Kiev, (1978), 143 pp.

[14] [14] Trost H. (1967), Auswirkungen des Superpositionsprinzips auf Kriech- und Relaxationsprobleme bei Beton und Spannbeton, *Beton-und Stahlbetonbau*, Vol. 62, (1967), No. 10, pp. 230-238; No. 11, pp. 261-269.

[15] [15] Rüsch H. and Jungwirth, D., Berücksichtigung der Einflüsse von Kriechen und Schwinden des Betons auf das Verhalten der Tragwerke, Werner Verlag, (1976), Düsseldorf.

[16] Šmerda Z., Křistek V., Creep and Shrinkage of Concrete Elements and Structures, *Elsevier, Amsterdam- Oxford- New York – Tokyo (1988), pp 296.*

[17] Zerna W., Trost H.: Rheologische Beschreibung des Werkstoffes Beton, Beton und Stahlbetonbau, Vol. 62, H.7, (1967), pp. 165–170.